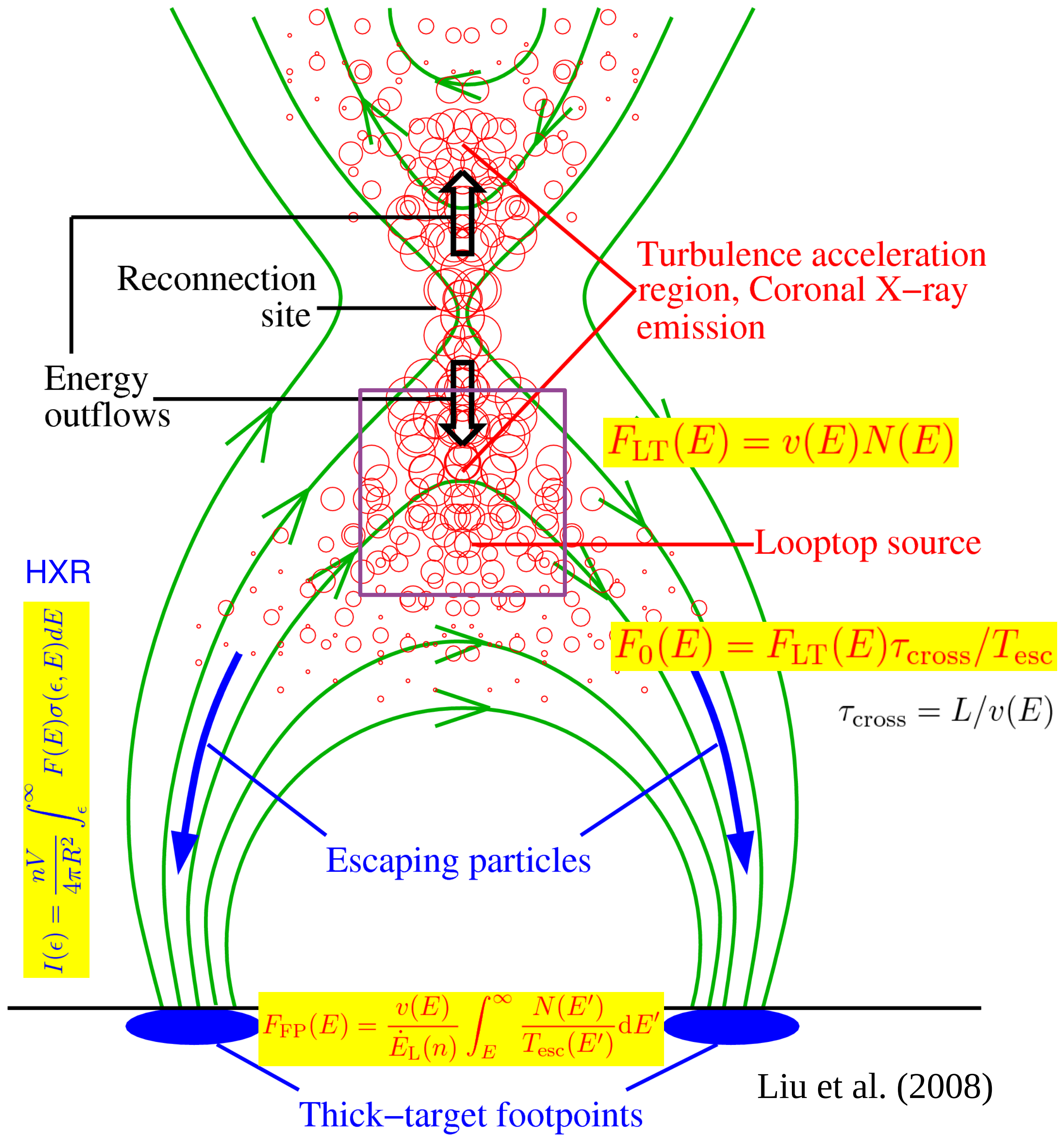
Accelerated Electron Spectra and Turbulence Characteristics from RHESSI Solar Flare Observations

Qingrong Chen and Vahé Petrosian Stanford University Physics Department (qrchen@gmail.com, vahep@stanford.edu)

In the stochastic acceleration (SA) model for solar flares, particles undergo simultaneous pitch angle scattering and acceleration by turbulence near the top of the flaring loop. The accelerated electrons produce HXR emission at the loop top (LT) and the footpoints (FPs) by bremsstrahlung. Thus imaging spectroscopic observations can directly give the accelerated electron spectrum at the LT, which is determined by the turbulence diffusion and direct acceleration rates, and can constrain some characteristics of turbulence. In particular, we can obtain the escape time and the pitch angle scattering time of electrons by turbulence from RHESSI flares using the regularized electron maps and compare the result with previous theoretical calculations. Second, we determine the turbulence direct acceleration rate by comparing the electron spectra obtained from solution of the steady-state Fokker-Planck equation with the directly observed LT electron spectra. Third, we demonstrate how to obtain the accelerated electron spectrum from the regularized inversion of the spatially integrated flare HXR spectrum for different escape times, which does not require imaging spectroscopy of spatially resolved spectra.

I. Stochastic Acceleration by Turbulence



$$F_{\mathrm{LT}}(E) = v(E)N(E)$$

$$F_0(E) = F_{\mathrm{LT}}(E)\tau_{\mathrm{cross}}/T_{\mathrm{esc}}$$

$$\tau_{\mathrm{cross}} = L/v(E)$$

$$I(\epsilon) = \frac{nV}{4\pi R^2}\int_\epsilon^\infty F(E)\sigma(\epsilon,E)dE$$

$$F_{\mathrm{FP}}(E) = \frac{v(E)}{\dot{E}_{\mathrm{L}}(n)}\int_E^\infty \frac{N(E')}{T_{\mathrm{esc}}(E')}\mathrm{d}E'$$

Liu et al. (2008)

The spectrum N(E) of electrons accelerated stochastically by turbulence can be described by the Fokker-Planck equation (Petrosian & Liu 2004).

$$\frac{\partial N(E)}{\partial t} = \frac{\partial^2}{\partial E^2}[D(E)N(E)] - \frac{\partial}{\partial E}\left[\left(A(E)-\dot{E}_{\mathrm{L}}(E)\right)N(E)\right] - \frac{N(E)}{T_{\mathrm{esc}}(E)} + \dot{Q}(E)$$

SA direct acceleration and diffusion rates	$A(E) = D(E)\zeta(E)/E + dD(E)/dE$ where $\zeta(E) = (2-\gamma^{-2})/(1+\gamma^{-1})$
Energy loss	$\dot{E}_{\mathrm{L}} = \dot{E}_{\mathrm{Coul}} = 4\pi r_0^2 m_{\mathrm{e}} c^3 n \ln\Lambda/\beta$
Escape time	$T_{\mathrm{esc}}(E) \simeq \tau_{\mathrm{cross}} + \tau_{\mathrm{cross}}^2/\tau_{\mathrm{scat}}$
Mean scattering time	$\tau_{\mathrm{scat}}(E) = \int_{-1}^{1} \frac{(1-\mu^2)^2}{D_{\mu\mu}^{\mathrm{Coul}}(\mu,E) + D_{\mu\mu}^{\mathrm{turb}}(\mu,E)} d\mu$
Coulomb scattering	$D_{\mu\mu}^{\mathrm{Coul}} = \frac{2(1-\mu^2)}{\gamma+1}\frac{\dot{E}_{\mathrm{Coul}}}{E}$

Accelerated electron flux spectrum at LT $F_{\mathrm{LT}}(E) = v(E)N(E)$

Effective radiating flux spectrum at FPs $F_{\mathrm{FP}}(E) = \frac{v(E)}{\dot{E}_{\mathrm{L}}(E)}\int_E^\infty \frac{N(E')}{T_{\mathrm{esc}}(E')}dE'$

Resulting bremsstrahlung HXR spectrum at LT and FPs

$$\left\{\begin{array}{c} I_{\mathrm{LT}}(\epsilon) \\ I_{\mathrm{FP}}(\epsilon) \end{array}\right\} = \frac{nV}{4\pi R^2}\int_\epsilon^\infty \left\{\begin{array}{c} F_{\mathrm{LT}}(E) \\ F_{\mathrm{FP}}(E) \end{array}\right\}\sigma(\epsilon,E)dE$$

II. Accelerated Spectrum, Escape Time & Turbulence Scattering Time from Regularized Electron Flux Images

Electron flux spectral images $F(x,y,E)$ are made available by regularized spectral inversion of the count visibilities in the Fourier domain (Piana et al. 2007). The resulting spatially resolved electron flux spectra at LT and FPs yields the accelerated electron spectrum $F_{\mathrm{LT}}(E)$, the escape time of electrons $T_{\mathrm{esc}}(E)$, and the pitch angle scattering time by turbulence $\tau_{\mathrm{scat}}^{\mathrm{turb}}(E)$ at the LT acceleration region (see Petrosian & Chen 2010).

$$T_{\mathrm{esc}}(E) = \frac{\tau_{\mathrm{L}}(E)(F_{\mathrm{LT}}/F_{\mathrm{FP}})}{\delta_{\mathrm{FP}}(E) + 2/(\gamma+\gamma^2)}$$

(collisional) energy loss time $\tau_{\mathrm{L}}(E) = E/\dot{E}_{\mathrm{L}}$

→ the mean and turbulence scattering times

$$\tau_{\mathrm{scat}} \simeq \tau_{\mathrm{cross}}^2/(T_{\mathrm{esc}} - \tau_{\mathrm{cross}}) \quad \text{and} \quad \tau_{\mathrm{scat}}^{\mathrm{turb}} \simeq \tau_{\mathrm{scat}}(1 + \tau_{\mathrm{scat}}/\tau_{\mathrm{scat}}^{\mathrm{Coul}})$$

→ the pitch angle scattering rate by turbulence $D_{\mu\mu}^{\mathrm{turb}}$

Example: 2003 November 3 flare (X3.9 class)

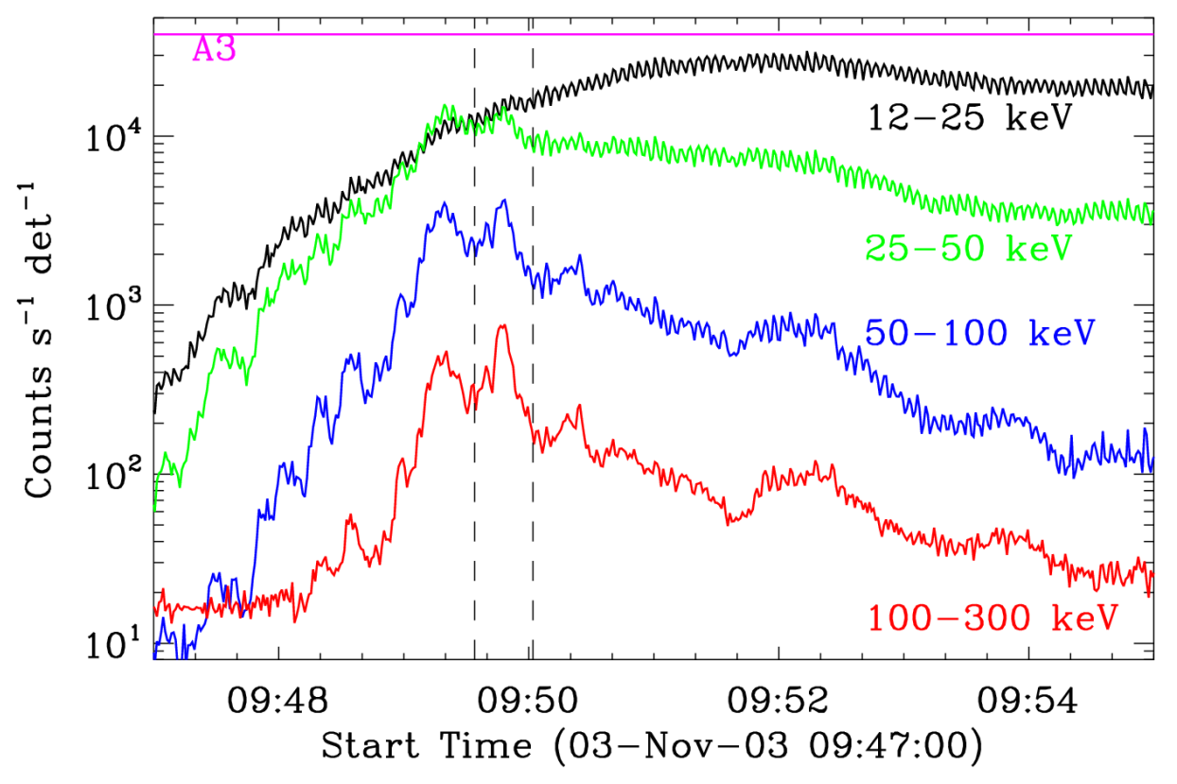


Figure 1. HXR count rate. The flare exhibits an unusually hard LT source up to 100-150 keV in HXR and two FPs during the impulsive phase (Chen & Petrosian 2011).

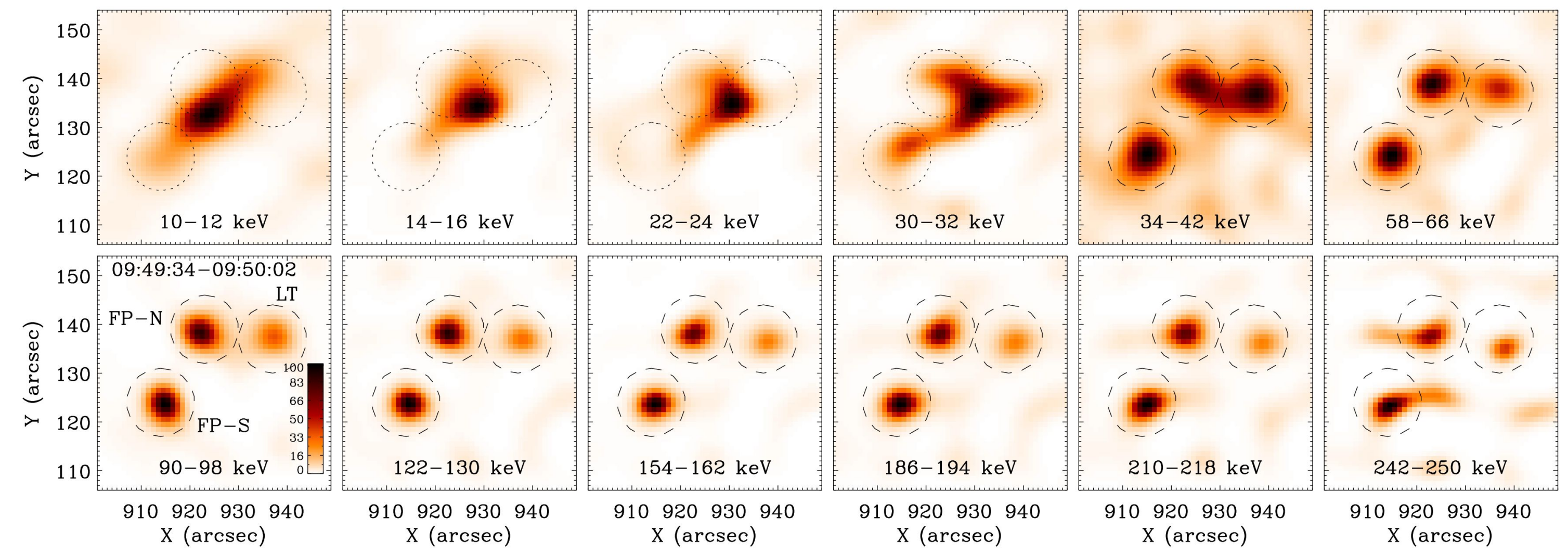


Figure 2. Electron flux spectral images up to 250 keV during the nonthermal peak as reconstructed from the regularized electron visibilities. The images show one LT and two FP sources above 34 keV and a loop structure at lower energies. The circles are used to extract fluxes for electron spectra.

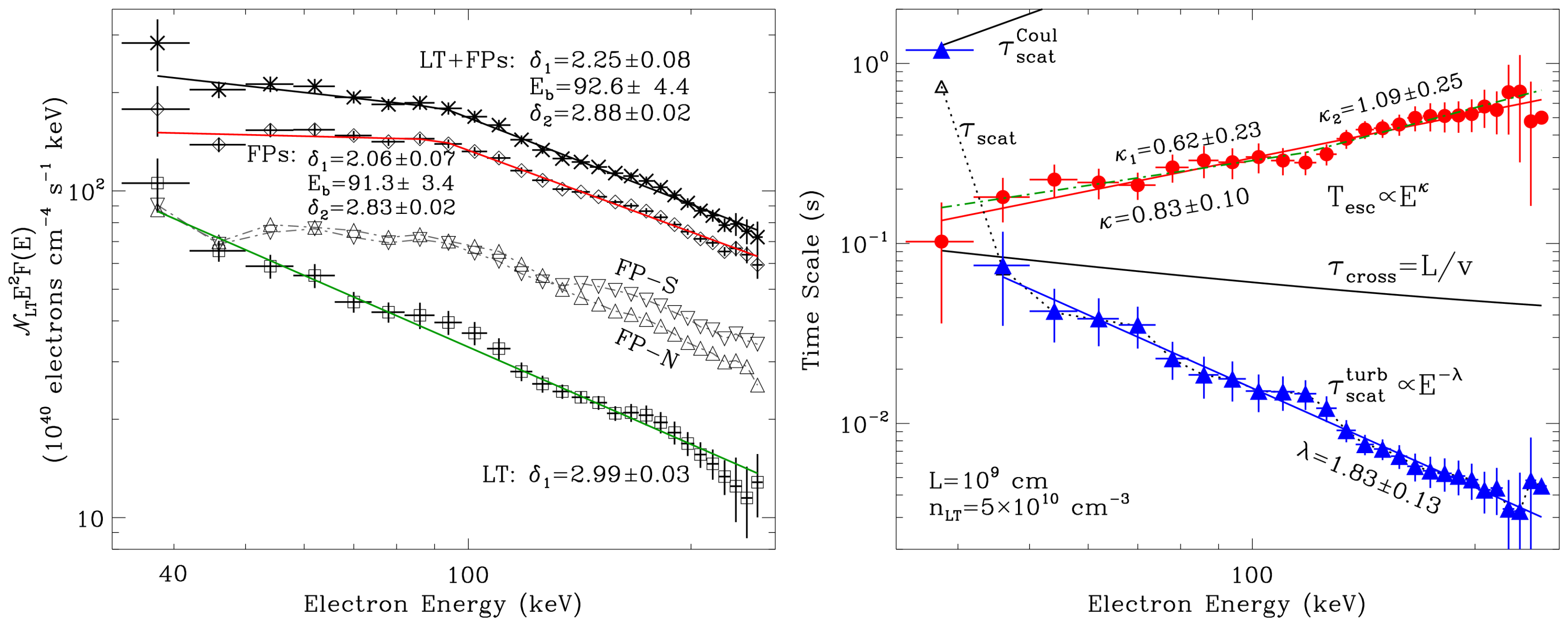


Figure 3. Left: Electron flux spectra for the **LT** source (square), the two **FP** sources summed (diamond), and all sources summed (cross). The LT spectrum can be fitted by a power law, and the summed FP and total spectra by a broken power law. Also shown are the spectra from the individual FP sources. Note that the LT and FP spectral difference is ~0-1, much smaller than commonly seen.
Right: ***Escape time*** (filled circle) and ***turbulence scattering time*** (filled triangular) of electrons in the acceleration region. The former can be well fitted by a power law or a broken power law increasing with electron energy, and the latter by a power-law rapidly decreasing with energy. Also shown are the crossing time, Coulomb scattering time, and the mean scattering time (open triangular).

III. Turbulence Direct Acceleration & Diffusion Time from Comparing with the Fokker-Planck Equation

Since the acceleration timescales are much shorter than the dynamic timescale (duration of the impulsive phase), we can simply consider the steady-state Fokker-Planck equation. In the above, we have determined the accelerated electron spectrum and the escape time as a function of electron energy between 30 and 250 keV. Therefore, in principle, one can determine the only unknown quantity in the Fokker-Planck equation, i.e. the direct acceleration rate *A(E)* or the diffusion rate *D(E)* by comparing the resulting spectrum with the observed accelerated spectrum. In the following we explore some possible forms of *A(E)* and *D(E)* in the Fokker-Planck equation that can accelerate a background thermal spectrum to the accelerated electron spectrum observed in the 2003 November 3 solar flare as presented above in Figure 3.

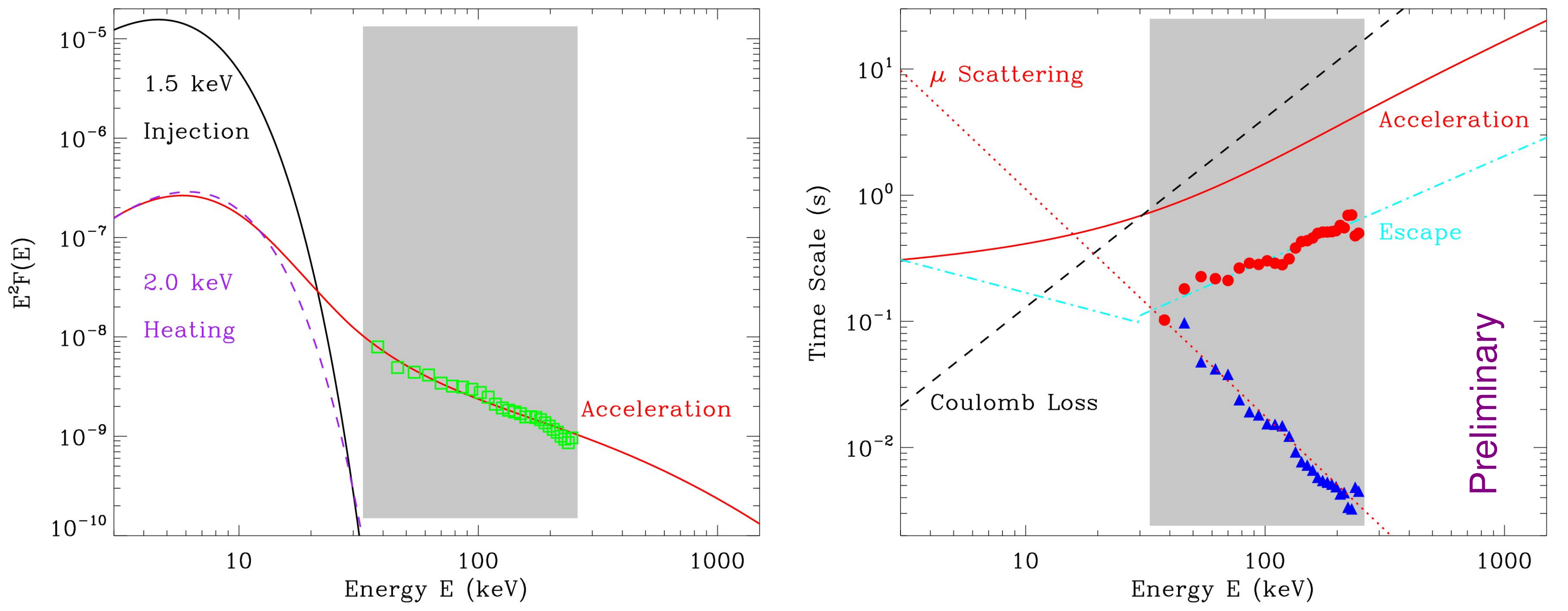


Figure 4. Left: Accelerated electron flux spectrum (red) from the solution (multiplied by velocity) of the steady-state Fokker-Planck equation under conditions specified in the right panel, superposed with the observed spectrum (square) in the 2003 November 3 solar flare. Note that acceleration is accompanied by heating. The shaded area represents the observed energy range from ~30-250 keV as in Figure 3.
Right: Timescale of direct acceleration (red, solid line) of electrons by turbulence, along with the assumed escape time (cyan, dash-dot) and Coulomb loss time as input to the Fokker-Planck equation. Also shown are the observed escape time (red, circles) and pitch angle scattering time (blue, triangular).

IV. Accelerated Spectrum from Spatially Integrated HXR spectrum

When imaging spectroscopy is not available, one can determine the accelerated electron spectrum from the spatially integrated HXR spectrum of the whole flaring loop, or equivalently, the total radiating electron spectrum.

$$F_{\mathrm{Tot}}(E) = F_{\mathrm{LT}}(E) + \frac{m_{\mathrm{e}}v^2}{E_{\mathrm{cr}}^2}\int_E^\infty \frac{F_{\mathrm{LT}}(E')}{T_{\mathrm{esc}}(E')/\tau_{\mathrm{cross}}(E')}dE'$$

Stopping energy due to collisional depth *nL*

$$E_{\mathrm{cr}} = \sqrt{2K(nL)},\ K = 2\pi r_0^2(m_{\mathrm{e}}c^2)^2\ln\Lambda$$

Therefore, if $F_{\mathrm{Tot}}(E)$ is known, one can solve for $F_{\mathrm{LT}}(E)$ from the above relation.

$$F_{\mathrm{LT}}(E) = F_{\mathrm{Tot}}(E) - \frac{m_{\mathrm{e}}v^2}{E_{\mathrm{cr}}^2}\int_E^\infty e^{\eta(E)-\eta(E')}\frac{F_{\mathrm{Tot}}(E')}{T_{\mathrm{esc}}(E')/\tau_{\mathrm{cross}}(E')}dE'$$

$$\eta(E) = \int_0^E \frac{m_{\mathrm{e}}v^2}{E_{\mathrm{cr}}^2}dE = \frac{1}{E_{\mathrm{cr}}^2}\frac{E^2}{(1+E/m_{\mathrm{e}}c^2)}$$

The collisional depth *nL* of the LT region and knowledge of the escape time are required for the above determination of the accelerated spectrum.

Note that the classical collisional thick target model assumes that the total HXR emission is produced by electrons *injected* somewhere in the corona into the loop. Thus

$$F_{\mathrm{Inj}}(E) = \frac{E_{\mathrm{cr}}^2}{(m_e v^2)E}\left(\frac{d\ln v^2}{d\ln E} - \frac{d\ln F_{\mathrm{Tot}}(E)}{d\ln E}\right)$$

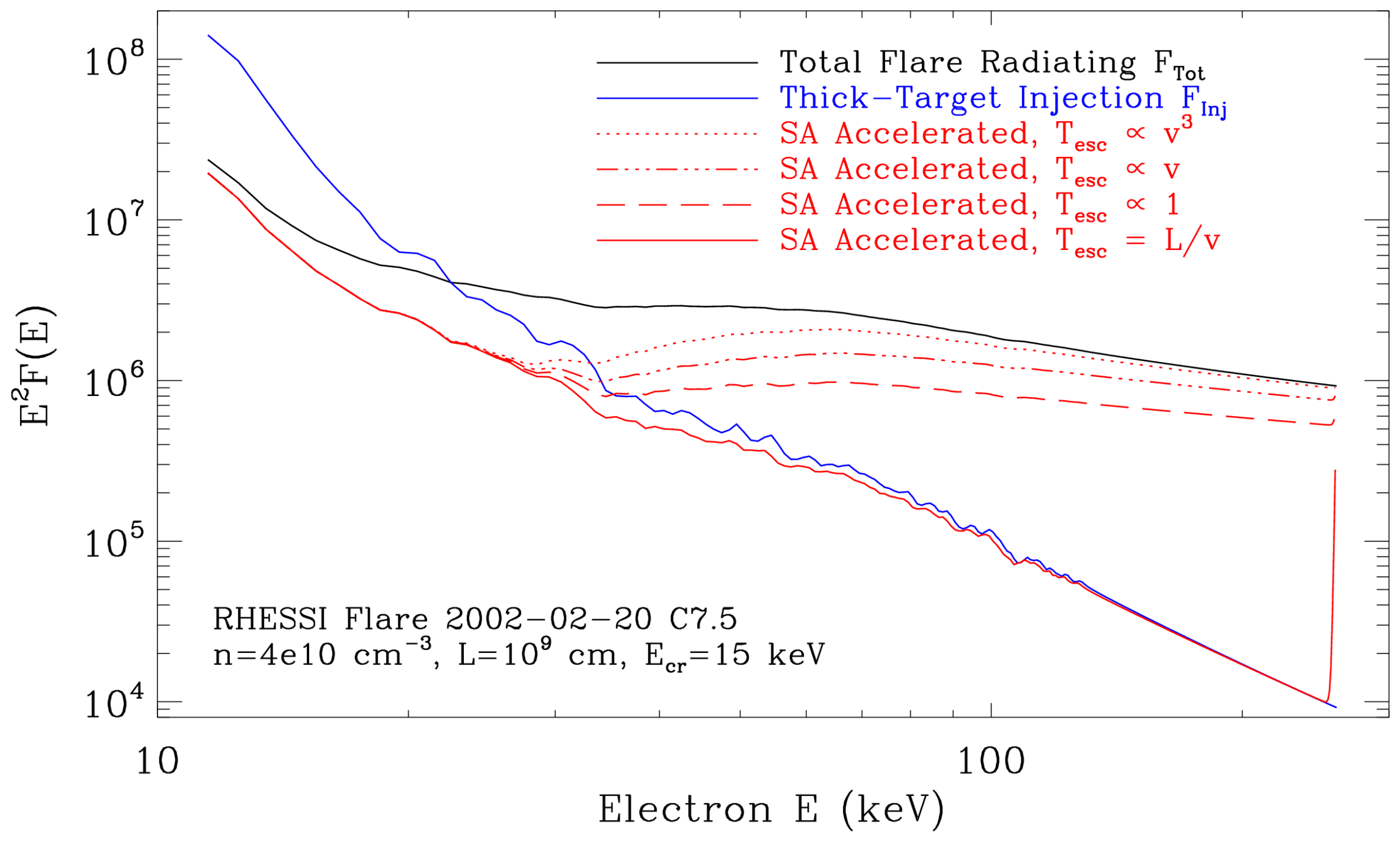
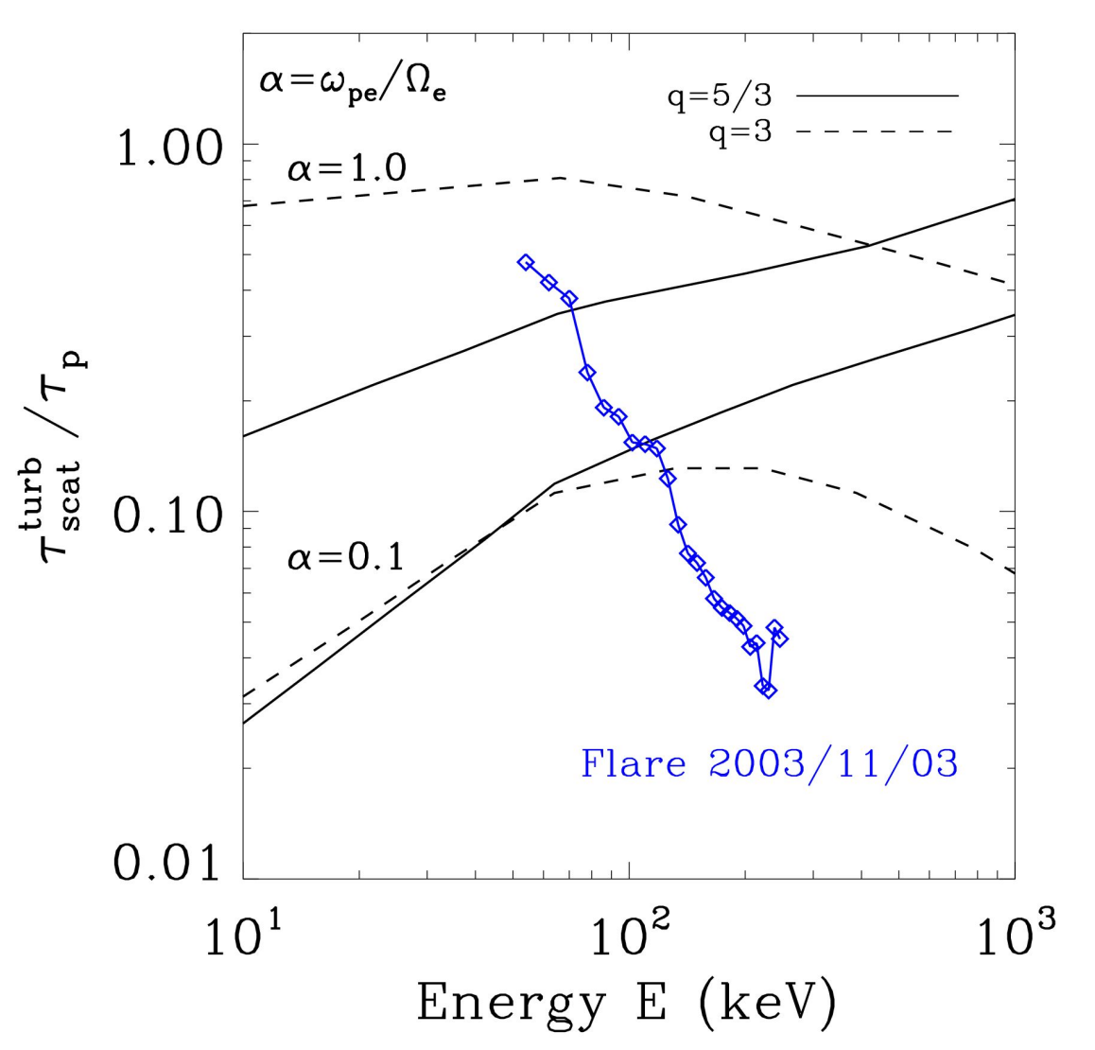


Figure 5. Determination of the accelerated electron spectrum (red) from the total radiating electron spectrum in the 2002 Feb 20 solar flare (C7.5 class) under four different escape times. When the escape time is same as the crossing time (i.e. electrons freely stream out of the LT region), the accelerated electron spectrum nearly matches the injected electron spectrum of the thick target model (blue) at high energies, both of which are softer than the total radiating spectrum by E^2. In the cases when the escape time becomes flatter above 30 keV compared to the crossing time (i.e. electrons are more confined at the LT region), the accelerated electron spectrum becomes harder and being closer to the total radiating spectrum at high energies, while it hardly changes at low energies.
Note that the total radiating electron spectrum is obtained by regularized inversion of the spatially integrated HXR spectrum (see Piana et al. 2003).

V. Summary and Discussion

In the SA model, particles are accelerated by turbulence at the LT region. The connection between the LT and FP spectra allows us to determine the energy dependence of the escape time and the pitch angle scattering time by turbulence for the accelerated electrons. We can then determine the direct acceleration and diffusion time due to turbulence by comparing the accelerated spectrum with the steady-state solution of the Fokker-Planck equation with input of the above escape time. Therefore, we may be able to determine the most important parameters for turbulence in solar flares, i.e. the pitch angle scattering rate and the direct acceleration and diffusion rate, based on the spatially resolved spectra from RHESSI's imaging spectroscopic observations of solar flares.
In the following, we present preliminary comparison of the above pitch angle scattering time with some theoretical calculation.

Figure 6. Theoretical calculation of the turbulence scattering time (Pryadko & Petrosian, 1997) for two values of the plasma parameter α and two values of the turbulence spectral index q. Here electrons are assumed to be accelerated by plasma waves propagating parallel to the large scale magnetic field lines.



As evident in the range 30-300 keV, the derived scattering time is much flatter than the observation. The rapidly decreasing scattering time seen in the 2003 November 3 flare may require a turbulence spectrum much steeper than the Kolmogorov or the Iroshnikov-Kraichnan spectrum. This is expected to be the case beyond the inertial range where damping is important.
On the other hand, inclusion of magnetic mirroring and anisotropic pitch angle distribution of the accelerated electrons may mitigate the above discrepancy.

Acknowledgment: We thank Anna Massone and Michele Piana for providing the electron visibility inversion code.

References:

Chen, Q. & Petrosian, V. 2011, in preparation
Liu, W. et al. 2008, ApJ, 676, 704
Petrosian, V. & Chen, Q. 2010, ApJL, 712, 131
Petrosian, V. & Liu, S. 2004, ApJ, 610, 550
Piana, M. et al. 2003, ApJL, 595, 127
Piana, M. et al. 2007, ApJ, 665, 846
Pryadko, J. M. & Petrosian, V. 1997, ApJ, 482, 774